1 Capacitors and DC Circuits

1.1 Formulas

Definition of capacitance: C = Q/V

Energy stored in a capacitor:

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Parallel plate capacitance w/o dielectric: $C = \epsilon_0 A/d$ Parallel plate capacitance w/ dielectric: $C = \kappa \epsilon_0 A/d$ Capacitors in series: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$ Capacitors in parallel: $C = C_1 + C_2 + \dots + C_n$ Definition of current: $i = \Delta Q/\Delta t$ Current and drift velocity: $I = nqv_d A$ Definition of resistance: $R = \Delta V/I$ Resistance of a wire: $R = \rho l/A$ Temperature variation of resistivity:

$$\rho = \rho_0 [1 + \alpha (T - T_0)]$$

Power dissipation in a resistor:

$$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

Steps: in application of Kirchhoff's Rules

- Label current: i_1, i_2, i_3, \ldots
- Node equation: $\sum i_{in} = \sum i_{out}$
- Loop equation: $\sum (\pm V) + \sum \mp iR = 0$

Resistors in series: $R = R_1 + R_2 + \cdots + R_n$

Resistors in parallel: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$

Charging in an RC circuit: $q(t) = Q(1 - e^{-t/RC})$

Discharging in an RC circuit: $q(t) = Qe^{-t/RC}$, where RC = τ is the time constant.

1.2 Electrons Through a Resistor Problem

If 5×10^{21} electrons pass through a 20Ω resistor in 10 min, what is the potential difference across the resistor? The fundamental charge is 1.602×10^{-19} C. Answer in units of V.

1.3 Voltage across a capacitor problem

Consider the capacitor network



What is the voltage across the 4 μ F (upper right hand) capacitor? Answer in units of V.

1.4 Unknown Capacitance Problem

When the switch is in position a, an isolated capacitor of unknown capacitance has been charged to a potential difference of 100 V. When the switch is moved to position b, this charged capacitor is then connected parallel to the uncharged 10 μ F capacitor. The voltage across the combination becomes 30 V.



Calculate the unknown capacitance. Answer in units of μ F.

1.5 Drift Velocity Problem

An aluminum wire with a cross-sectional area of 4×10^{-6} m² carries a current of 5 A.

Find the drift speed of the electrons in the wire. Assume that each atom supplies one electron. Aluminum has a molecular weight of 26.98 g/mol and a density of 2.7 g/cm³. Avogadro's number is 6.022×10^{23} and the fundamental charge is 1.602×10^{-19} C. Answer in units of m/s.

1.6 Equivalent Resistance Problem

Four resistors are connected as shown in the figure.



Find the resistance between points a and b. Answer in units of Ω .

1.7 Find R in Circuit with Switch Problem

In the circuit shown below, the current i in the resistor ${\cal R}$ doubles its original value when the switch ${\cal S}$ is closed.



Find the value of R. Answer in units of Ω .

1.8 Charge on Capacitor Problem

In the figure below the battery has an emf of 4 V and an internal resistance of $1\Omega.$ Assume there is a steady current flowing in the circuit.



Find the charge on the 2 $\mu {\rm F}$ capacitor. Answer in units of $\mu {\rm C}.$

1.9 Voltage across Capacitor Problem

The circuit has been connected as shown in the figure for a "long" time.



What is the magnitude of the electric potential across the capacitor? Answer in units of V.

1.10 Two Loop Circuit Problem

Consider the circuit



Find i_1 . Answer in units of A.